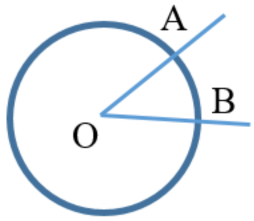
Formulių lapas: Trigonometrija

Laipsnių ir radianų sąryšis

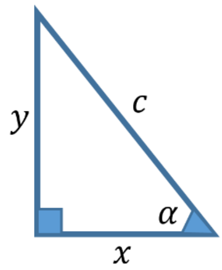


Pakeičiant laipsnius į radianus, naudojama formulė: $rad = \frac{\pi}{180} \cdot \alpha$

Pakeičiant radianus į laipsnius, naudojama formulė: $\alpha = \frac{180}{\pi} \cdot rad$

$$1rad = \frac{\smile AB}{OB} = \frac{180}{\pi} = 57^0 17'45''$$

Trigonometrija stačiajame trikampyje



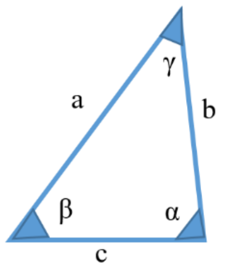
$sin\alpha = \frac{y}{c}$ $\quad cos\alpha = \frac{x}{c}$ $\quad tg\alpha = \frac{y}{x}$ $\quad ctg\alpha = \frac{x}{y}$

Trigonometrija įvairiakraščiame trikampyje

Sinusų teorema: $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = 2R$

Kosinusų teorema: $a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

Ploto formulė: $S = \frac{1}{2} bc \cdot \sin\alpha$



Trigonometrinių funkcijų lentelė

Laipsniai	0	30	45	60	90	180	270	360
Radianai	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\mathrm{tg}\,\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-	0
$\mathrm{ctg}\,\alpha$	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	-	0	-

Trigonometrinių funkcijų periodas

$sinx \ ir \ cosx$ mažiausias teigiamas periodas yra 2π, o $\sin(nx) \ ir \ \cos(nx)$ yra $T = \frac{2\pi}{n}$

$tgx \ ir \ ctgx$ mažiausias teigiamas periodas yra π, o $tg(nx) \ ir \ ctg(nx)$ yra $T = \frac{\pi}{n}$

Trigonometrinių funkcijų lyginumas

$\sin(-\alpha) = -\sin\alpha$	$arcsin(-\alpha) = -arcsin(\alpha)$
$\cos(-\alpha) = \cos\alpha$	$\arccos(-\alpha) = \pi - arccos(\alpha)$
$\mathrm{tg}(-\alpha) = -\mathrm{tg}\,\alpha$	$\mathrm{arctg}(-\alpha) = -\mathrm{arctg}(\alpha)$
$\mathrm{ctg}(-\alpha) = -\mathrm{ctg}\,\alpha$	$\mathrm{arcctg}(-\alpha) = \pi - \mathrm{arcctg}(\alpha)$

Pagrindinės trigonometrinės formulės

$$(\sin\alpha)^2 + (\cos\alpha)^2 = 1 \qquad \operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$$

$$1 + (\operatorname{tg}\alpha)^2 = \frac{1}{(\cos\alpha)^2} \quad 1 + (\operatorname{ctg}\alpha)^2 = \frac{1}{(\sin\alpha)^2} \quad \operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha = 1$$

Trigonometrinių funkcijų argumentų sumos ir skirtumo formulės

$$\sin(\alpha \pm \beta) = \sin\alpha\cdot\cos\beta \pm \cos\alpha\cdot\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cdot\cos\beta \mp \sin\alpha\cdot\sin\beta$$

$$tg(\alpha \pm \beta) = \frac{tg\alpha \pm tg\beta}{1 \mp tg\alpha\cdot tg\beta} \qquad ctg(\alpha \pm \beta) = \frac{ctg\alpha\cdot ctg\beta \mp 1}{ctg\alpha \pm ctg\beta}$$

Trigonometrinių funkcijų sandaugos ir keitimo suma formulės

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$$

$$\sin\alpha\cdot\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

Trigonometrinių funkcijų sumos ir skirtumo formulės

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

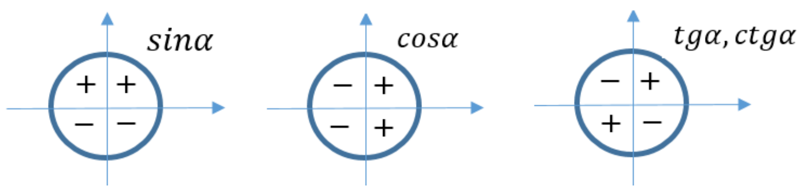
Trigonometrinių funkcijų dvigubo kampo ir laipsnio žeminimo formulės

$$\sin(2\alpha) = 2\sin\alpha\cdot\cos\alpha \qquad \cos(2\alpha) = (\cos\alpha)^2 - (\sin\alpha)^2 \qquad tg(2\alpha) = \frac{2tg\alpha}{1-(tg\alpha)^2}$$

$$(\sin\alpha)^2 = \frac{1}{2}(1-\cos 2\alpha) \quad (\cos\alpha)^2 = \frac{1}{2}(1+\cos 2\alpha)$$

Redukcijos lentelė ir ketvirčių ženklai

	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
	$90-\alpha$	$90+\alpha$	$180-\alpha$	$180+\alpha$	$270-\alpha$	$270+\alpha$	$360-\alpha$
$\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$
$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$

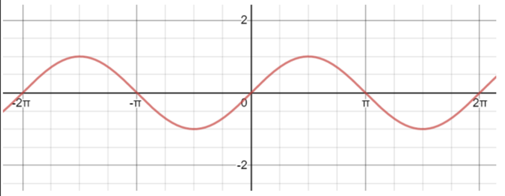


Trigonometrinių lygčių sprendimas

$\sin x = m$		$\cos x = m$	
$\sin x = m, [m] \leq 1$	$x = (-1)^k \arcsin(m) + \pi k,\ k\epsilon Z$	$\cos x = m, [m] \leq 1$	$x = \pm\arccos(m) + 2\pi k,\ k\epsilon Z$
$\sin x = 0$	$x = \pi k,\ k\epsilon Z$	$\cos x = 0$	$x = \frac{\pi}{2} + \pi k,\ k\epsilon Z$
$\sin x = 1$	$x = \frac{\pi}{2} + 2\pi k,\ k\epsilon Z$	$\cos x = 1$	$x = 2\pi k,\ k\epsilon Z$
$\sin x = -1$	$x = -\frac{\pi}{2} + 2\pi k,\ k\epsilon Z$	$\cos x = -1$	$x = \pi + 2\pi k,\ k\epsilon Z$

$$\operatorname{tg} x = m,\ x = \operatorname{arctg}(m) + \pi k,\ k\epsilon Z$$

Trigonometrinių funkcijų grafikai

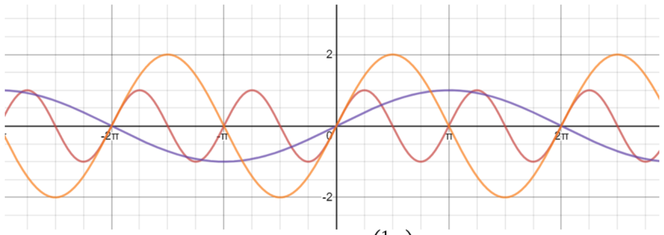


$\mathbf{y = \sin x}$

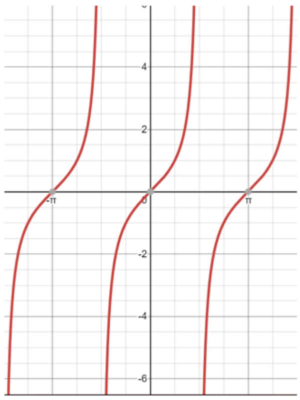
$D_f = (-\infty; +\infty)$

$E_f = [-1; 1]$

$T = 2\pi$



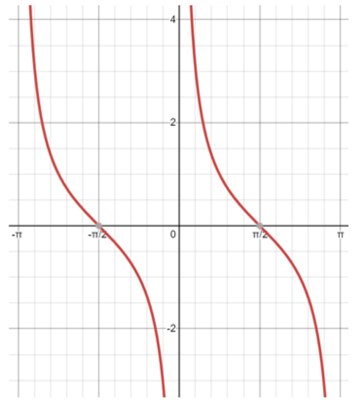
■ $y = \sin(2x)$ ■ $y = \sin\left(\frac{1}{2}x\right)$ ■ $y = 2\sin(x)$



$\mathbf{y = tg\, x}$

$D_f = \left(-\frac{\pi}{2}; +\frac{\pi}{2}\right)$ $E_f = (-\infty; +\infty)$

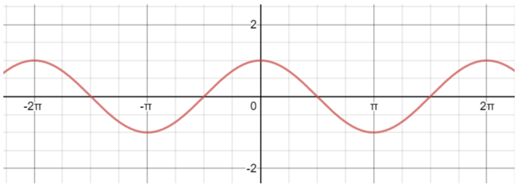
$T = \pi$



$\mathbf{y = ctg\, x}$

$D_f = (0; \pi)$ $E_f = (-\infty; +\infty)$

$T = \pi$

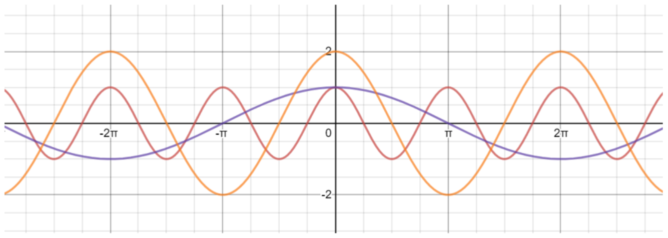


$\mathbf{y = \cos x}$

$D_f = (-\infty; +\infty)$

$E_f = [-1; 1]$

$T = 2\pi$



■ $y = \cos(2x)$ ■ $y = \cos\left(\frac{1}{2}x\right)$ ■ $y = 2\cos(x)$

■ $\boldsymbol{y = arcsinx}$

$D_f = [-1; 1]$ $E_f = \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

■ $\boldsymbol{y = arccosx}$

$D_f = [-1; 1]$ $E_f = [0; \pi]$

■ $\boldsymbol{y = arctgx}$

$D_f = R$ $E_f = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

■ $\boldsymbol{y = arcctgx}$

$D_f = R$ $E_f = (0; \pi)$